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STABILITY OF NONISOTHERMAL FLOW OF A VISCOUS LIQUID IN AN ANNULAR CHANNEL

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In nonisothermal flow of a viscous liquid in an annular channel between coaxial cylinders where the outer cylinder has finite dimensions and is stationary, and the inner cylinder infinitely moves along the axis, the central position of the latter is unstable. When superimposing a thermal field, principally it is possible to create as large a force as required which holds the inner cylinder exactly on center.

Two coaxial cylinders with a liquid between them are considered. A similar system is often met with in various fields of engineering [1, 2]. In particular, they are a matter of practical interest in technology of polymer coatings when the inner infinite cylinder (fiber) moves in the direction of its axis and the outer cylinder of finite dimensions acts as a gauging device. A polymer solution is fed to the gauge inlet, and at the outlet this solution is entrained by the fiber forming a polymer coating on it. The important parameters of such a process are the thickness of a polymer film on the fiber and its uniformity.

In this problem the gap between the inner and outer cylinders may act as a perturbation which makes it possible to obtain, by approximate methods, analytical expressions for characterizing the system.

1. We consider the general case of nonisothermal stationary nonaxisymmetric motion of a viscous liquid in an annular channel between cylinders when the outer one has a cone shape. To simplify the problem, we make the usual assumptions of developed flow [3, 4]. The liquid viscosity is dependent on temperature [5]. In the final analysis we consider the system of equations

$$\frac{\partial P}{\partial r} = 0; \tag{1}$$

$$\frac{\partial P}{\partial z} = \mu_0\left(\frac{\partial^2 V_z}{\partial r^2} + \frac{\partial V_z}{r\partial r} - \frac{k}{T_0}\frac{\partial T}{\partial r}\frac{\partial V_z}{\partial r}\right); \tag{2}$$

$$\frac{\partial P}{r\partial\varphi} = \mu_0\left(\frac{\partial^2 V_\varphi}{\partial r^2} + \frac{\partial V_\varphi}{r\partial r} - \frac{V_\varphi}{r^2} - \frac{k}{T_0}\frac{\partial T}{\partial r}\left(\frac{\partial V_\varphi}{\partial r} - \frac{V_\varphi}{r}\right)\right); \tag{3}$$

$$\frac{\partial V_z}{\partial z} + \frac{\partial V_r}{\partial r} + \frac{V_r}{r} + \frac{\partial V_\varphi}{r\partial\varphi} = 0; \tag{4}$$

$$c\left(V_z\frac{\partial T}{\partial z} + V_r\frac{\partial T}{\partial r}\right) = \lambda\left(\frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{r\partial r}\right) \tag{5}$$

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with the boundary conditions

$$r = R_1(\sqrt{1-(\alpha\sin\varphi)^2} + \alpha\cos\varphi);\; V_z = U;\; V_\tau = 0;\; V_r = 0;\; T = T_1(z); \tag{6}$$

$$r = R_2(z) = R_{21} - (R_{21} - R_{22})y;\; V_z = V_r = V_\varphi = 0;\; T = T_2(z); \tag{7}$$

$$z = 0;\; P = P_1;\; T_1 = T_0; \tag{8}$$

$$z = L;\; P = P_2. \tag{9}$$

Here y = z/L, α is the distance between the fiber and gauge axes (in relative units).

For the system under consideration the most available in experiments is the value of velocity of passage of the liquid through the annular channel

$$Q = 2\int_0^\pi\int_{R_1}^{R_2} V_z r dr d\varphi, \tag{10}$$

which can characterize other parameters of the system, which are of practical interest but are difficult for experimental observation. The value of hydrodynamic force important for the present work as affecting the surface of the inner cylinder is classified among the latter:

$$F = -2R_1\int_0^L\int_0^\pi P(\cos\varphi + \alpha\cos 2\varphi)\,dz d\varphi. \tag{11}$$

2. We will start analyzing the system with the isothermal variant T = const. Here it is acceptable to take $V_r = 0$.

The procedure of solution is as follows. From (2), (3) with the boundary conditions (6), (7) we find an expression for V_z, substitute it into (4) and, solving the latter with the boundary conditions (8), (9), obtain

$$P = P_0\left\{\frac{1}{\gamma(\Delta_1+\Delta_2)}\left[(1-y)\left(\beta y + \frac{P_1}{P_0}\Delta_1(\Delta_2+\gamma)\right) + \frac{P_2}{P_0}\Delta_2^2 y(\Delta_1+\gamma)\right] + \right.$$
$$+\frac{2\alpha\beta y(1-y)(\Delta_1+\Delta_2+\gamma)}{3\gamma^3(\Delta_1+\Delta_2)^2}\left(1-\Delta_1\Delta_2\frac{P_2-P_1}{P_0}\right)\times \tag{12}$$
$$\left.\times\left[1-\frac{2}{\pi}\left(\varphi+\frac{3\alpha}{\gamma}\sin\varphi\right)\right]\right\}.$$

Here we have introduced the notation

$$P_0 = \frac{4\mu UL}{R_1^2};\; \beta = \frac{R_{21}-R_{22}}{R_1};\; \gamma = \Delta - \beta y;$$
$$\Delta_1 = \frac{R_{21}-R_1}{R_1} < 1;\; \Delta_2 = \frac{R_{22}-R_1}{R_1} < 1.$$

In accordance with (11) we obtain the expression for the force affecting the fiber, i.e., centering:

$$F_c = -\frac{8\alpha\beta P_0 L R_1}{3\pi\Delta_1\Delta_2(\Delta_1+\Delta_2)^2}\left(1+\frac{2\alpha^2(\Delta_1^2+2\Delta_1\Delta_2-\Delta_2^2)}{3\Delta_1^2\Delta_2}\right)\left(1-\Delta_1\Delta_2\frac{P_2-P_1}{P_0}\right). \tag{13}$$

It is convenient to carry out further analysis, taking $P_2 = P_1$ for simplification. The optimal value of the gauge taper β_0 when $|F_{0c}|$ = max, will be

$$\beta_0 = \frac{3}{4}\Delta_2. \tag{14}$$

The maximum centering force is

$$F_{0c} = -\frac{\mu U L^2 R_1}{5(R_{21}-R_1)^2}. \tag{15}$$

The order of values for the typical case of enamel production [3]: $L = 10^{-2}$ m; $\mu = 1$ N·sec/m^2; $U = 1$ m/sec; $R_1 = 10^{-4}$ m; $R_{21} - R_1 = 2\cdot10^{-5}$ m; $F_{0c} = -5$ N.

3. We will investigate the stability problem of the central position of a fiber by common methods of the theory of stability of viscous liquid motion [6]. The pressure with nonaxisymmetric perturbation, fitting the basic system (1)-(5) (T = const) and preserving the type of liquid motion, will be

$$P_* = P + \Delta P, \tag{16}$$

where P is described by Eq. (12),

$$\Delta P = \frac{2\alpha y}{3\gamma^3(\Delta_1+\Delta_2)}\left[(\Delta_1^2+\Delta_1\gamma+\gamma^2)\left(\frac{\Delta_2}{\Delta_1}\right)^2+\beta(1-y)\frac{\Delta_1+\Delta_2+\gamma}{\Delta_1+\Delta_2}\right]\times \left(1-\Delta_1\Delta_2\frac{P_2-P_1}{P_0}\right)\left(1-\frac{2}{\pi}\left(\varphi+\frac{3\alpha}{\gamma}\sin\varphi\right)\right). \tag{17}$$

From (11) we find the force affecting the fiber under perturbation:

$$F_* = \frac{8\alpha P_0 L R_1}{3\pi\Delta_1^2(\Delta_1+\Delta_2)}\left\{1+\frac{\Delta_1}{2\Delta_2}+\frac{\alpha^2}{6\Delta_1}\left[1+\frac{\beta}{\Delta_1}\left(5+4\frac{\Delta_1}{\Delta_2}\right)+ +2\left(\frac{\Delta_1}{\Delta_2}+\frac{\Delta_2}{\Delta_1}\right)^2\right]\right\}\left(1-\Delta_1\Delta_2\frac{P_2-P_1}{P_0}\right). \tag{18}$$

Comparing F_c with F_* under the simplifying assumption $P_2 = P_1$ yields

$$\left|\frac{F_*}{F_c}\right| = (\Delta_1+\Delta_2)\frac{\Delta_1+2\Delta_2}{\beta\Delta_1}. \tag{19}$$

For the gauge with optimal taper when $\beta = 3\Delta_2/4$; $\Delta_1 = 7\Delta_2/4$, we obtain $|F_*/F_c| \approx 8$. Here $F_* > 0$, i.e. this force is anti-centering and exceeds the centering force by an order. Thus, the optimal gauge shape cannot ensure the central position of a fiber. This leads to a nonuniform thickness of the polymer coating in the fiber cross section.

4. Now we will study a nonisothermal system of coaxial cylinders ($R_{21} = R_{22} = R_2$) whose temperature vary along the axis and the liquid viscosity depends on temperature [5]:

$$\mu = \mu_0 \exp\left(-k\frac{T-T_0}{T_0}\right). \tag{20}$$

We will analyze the system (1)-(5) with the boundary conditions (6)-(9) for two limiting cases:

1) heat conduction described by the right side of Eq. (5) dominates;

2) forced convection described by the left side of Eq. (5) dominates.

We will make the simplifying assumptions:

the function $T_2(y)$ is linear:

$$T_2(y) = T_0(1+ay), \tag{21}$$

where

$$a = k\frac{T_0 - T_2(1)}{T_0 \ln x_2}; \quad x_2 = \frac{R_2}{R_1} - \beta y;$$

the function T_1 is the constant value.

Given below is the result of solving the axisymmetric problem ($\alpha = 0$) for case 1):

$$T = (T_2 - T_0)\frac{\ln x}{\ln x_2} + T_0; \tag{22}$$

$$Q = \pi U R_1^2 \Delta \left[1 + \frac{\Delta}{2(a^2 - 12)} \left(a^2 + \frac{3}{2} a - 6 + \frac{3R_1^2}{\mu_0 U L} (P_2 - P_1) \right) \right], \tag{23}$$

where

$$\Delta_1 = \Delta_2 = \Delta; \quad x = \frac{r}{R_1}.$$

From the analysis of Eq. (23) it is evident that for the critical value of temperature drop on the gauge ends, when $a = -3.5$, the value of $Q = 0$, i.e. flow through the gauge ceases. The "temperature choking" of the gauge takes place.

The other limiting case – the domination of convection – is characteristic of large velocities of the process. By solving we obtain the expression describing the liquid temperature in the channel:

$$T = T_0 + (T_2 - T_0) \frac{\ln(1+g)}{\ln(1+g_2)}, \tag{24}$$

where

$$g = \frac{x-1}{q - x_2 - 1}; \quad g_2 = \frac{x_2 - 1}{q - x_2 - 1}; \quad q = \frac{T_*}{T_2 - T_0};$$

T_* is the constant characterizing the liquid from the viewpoint of occurrence in the system of "critical phenomena" [5].

These critical phenomena occur in the channel cross section with the coordinate $z = z_*$, satisfying the equation

$$T_2(z_*) = T_0 + \frac{T_*}{x_2 - 1}. \tag{25}$$

For the assumed linear function $T_2(z)$

$$z_* = \frac{LT_*}{aT_0(x_2 - 1)}. \tag{26}$$

To avoid the occurrence of the "critical phenomenon" it is necessary to satisfy the condition

$$T_* \geqslant 4.83 (x_2 - 1) T_0 \Delta. \tag{27}$$

Under convection the use of the function $T_2(z)$ as a governing parameter is ineffective since the value of Q does not practically differ from the isothermal case.

5. In order to find out if it is possible to center a fiber in the gauge we go back to the variant of dominant heat conduction (case 1). For this purpose we solve the system (1)-(4), (6)-(9), when $R_{21} = R_{22} = R_2$.

For finding the function $P(y, \varphi)$ we integrate Eq. (4) for x going from x_1 to x_2. The solution of the obtained equation yields

$$P = P_1 + (P_2 - P_1) y - \frac{\mu_0 \alpha U \tau_{20}}{L \Delta^4} \left[1 - \frac{\tau_2}{\Delta} - \left(1 - \frac{\tau_{20}}{\Delta} \right) \exp\left(2 \frac{y-1}{\Delta_3} \right) - \exp\left(-\frac{2y}{\Delta_3} \right) \right] \cos\varphi, \tag{28}$$

where

$$\tau_2 = k \frac{T_2(z) - T_0}{T_0}; \quad \tau_{20} = k \frac{T_2(L) - T_0}{T_0}; \quad \Delta_3 = \frac{R_1}{L} < 1.$$

From (11) we obtain the final expression for the centering force:

$$F_c = \frac{\pi \alpha \mu_0 \tau_{20} U R_1}{\Delta^4 (\Delta + 0.5\tau_{20})}. \tag{29}$$

From the analysis of this formula it follows that, by choosing the value of τ_{20} (temperature drop on the gauge ends) under heat conduction it is possible to obtain as large a centering force as required. Therefore, it is possible to obtain any desired degree of concentricity of a polymer coating on a fiber.

For evaluating the value of a centering force we will take parameters related to the case of enamel production with polyurethane lacquers [3]. In the temperature range 20-100°C the temperature constant is $k = 3.45$ which corresponds to at least a twofold change in the lacquer viscosity. On the gauge 2 cm long creating a temperature drop about 40°C is quite realizable. Then at the initial temperature of lacquer $T_0 = 350$ K (77°C) we obtain $\tau_{20} = -0.398$. For the rest of the quantities we take values given in Para. 2. The value of the centering force calculated by Eq. (29) will be $F_c = -62$ N. It is 1.5 times as large as the anticentering force ($F_* = 40$ N) and thus will be sufficient for obtaining a coating with a high degree of concentricity.

6. Analytical expressions are obtained for the force centering a fiber in a gauge and for the velocity of liquid passage through the gauge in isothermal and nonisothermal flow. The instability of the central position of the fiber is shown not only in a cylindrical but also in a cone gauge in isothermal flow. In the nonisothermal case for the corresponding temperature drop along the gauge length it is possible to obtain as large a centering force as required. At large velocities of the process, however, a shift from laminar to turbulent flow may take place. The occurrence criterion of this critical phenomenon is found. The expressions for a centering force can be applied in designing plants for optical fiber extraction [1], and those for velocity of liquid passage through a gauge – in production of enamel wires [2]. The dependence of the velocity of passage of a viscous liquid on the temperature drop along the gauge length can be used in various processes. The temperature drop is an effective governing parameter of the process.

NOTATION

r, z, φ, coordinates of cylindrical system fixed at the gauge axis; V_r, V_z, V_φ, velocity vector components of liquid; P, P_1, P_2, pressure of liquid in channel, at the outlet and inlet of gauge, respectively; P_*, pressure in the presence of perturbation; T, T_0, liquid temperature in channel and at the inlet to channel; $T_1(z)$, $T_2(z)$, temperature of the inner and outer cylinders, respectively; T_*, constant, characterizing liquid; μ, dynamic viscosity of liquid; μ_0, viscosity at T_0; k, constant of liquid; c, specific heat capacity per unit volume; λ, thermal conductivity; U, velocity of fiber motion along the axis with reference to gauge; R_1, radius of fiber; R_2 inner radius of gauge; R_{21}, R_{22}, radii of the inlet and outlet of gauge, respectively; L, gauge length; α, displacement of fiber axis with reference to gauge axis; Q, velocity of passage of liquid through annual channel in gauge; F, force affecting fiber surface in gauge; F_c, centering force; F_{0c}, maximal centering force; F_*, anticentering force.

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